Thursday, March 22, 2018 3:28 PM

Machine Scheduling

Problem Setting

m machines

n jobs $\rightarrow$ job j has a processing time of P_j

$$P_j \in \mathbb{Z}_{\geq 0}$$

Goal: Schedule / Assign jobs to machines to minimize the max. load over all machines

(equivalently)

min max completion time over all jobs

Simple Greedy Algo

Sort jobs in descending order of sizes & send each job to least loaded machine

$\downarrow$

how good is this algorithm?

Theorem:-

If OPT makespan is T^*, then the above greedy has makespan $\leq \frac{4}{3} T^*$

Even dumber algorithm:-

No sorting

look at all jobs in arbit. order

send each job to least loaded M/C.

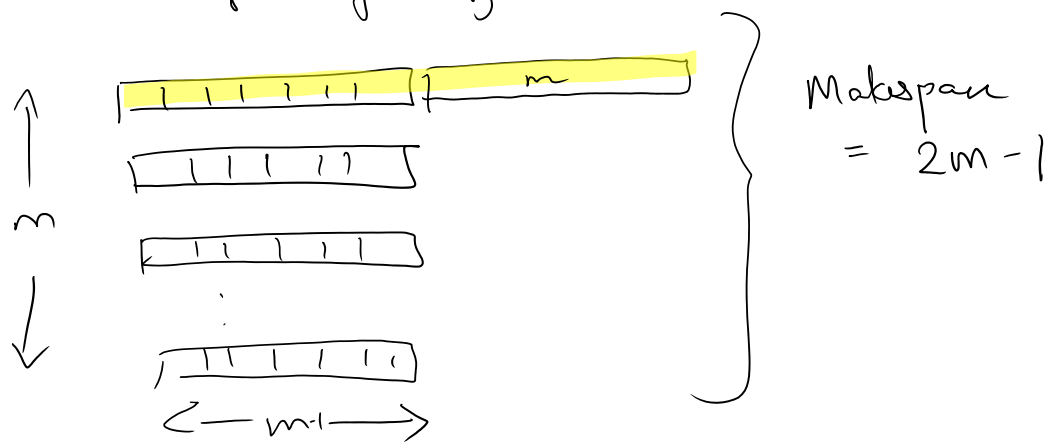
Q: How good is this algo??

Thm: Algo is 2-approximation

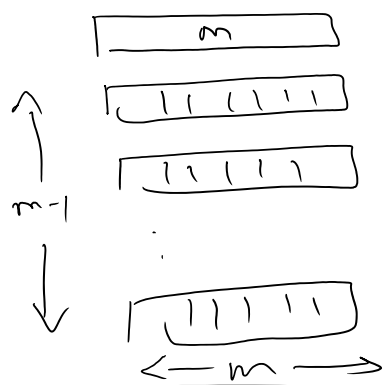
Suppose $n = m(m-1)+1$

1st $m(m-1)$ jobs have $p_j = 1$

1 job of $p_j = m$



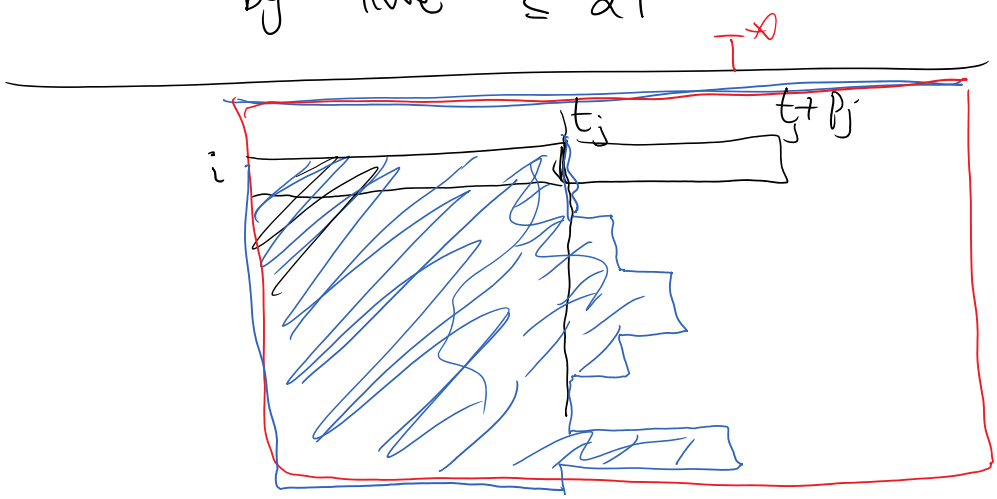
OPT



OPT makespan $= m$

Proof:

If OPT makespan $= T^*$, each job j <u>completes</u> by time $\leq 2T^*$



<u>Lem</u>: If job j starts @ time t_j & ends at time $t+p_j$, then all machines are <u>busy</u> for the 1st t time units

$$\sum_{\text{all } j'} p_{j'} \geq \sum_{\substack{\text{all jobs} \\ j' \text{ before} \\ j}} p_{j'} \geq m \cdot t_j$$

$$\Rightarrow t_j \leq \frac{\sum_{\text{all jobs}} p_j}{m} \leq T^*$$

$$\sum_{\text{all jobs } j} p_j \leq m T^*$$

$\Rightarrow$ Starting time of job $j \leq T^*$

$\Rightarrow$ Ending time of job $j \leq T^* + p_j$
$\leq 2T^*$

$$\boxed{\begin{aligned} T^* &\geq p_j \quad \forall j \\ T^* &\geq \frac{\sum p_j}{m} \end{aligned}}$$

$\leftarrow$ LB bounds are used in the <u>proof</u>.

<u>Thm</u>:
There exists a $(1+\varepsilon)$-approximation to the makespan with running time $f(n, m, \frac{1}{\varepsilon})$ $\leftarrow$ for any fixed $\varepsilon \geq 0$

this is a poly(m, n)

e.g.: $(m+n)^{1/\varepsilon}$ $\longrightarrow$ PTAS algorithm

e.g. $(m+n)$ $\longrightarrow$ <u>PTAS</u> algorithm

Poly time Approximation Scheme

<u>Idea: Dynamic Programming</u> !!

Suppose there are only C (constant) many types of jobs

n_1	t_1	p_{j_1}	$p_j = 1$	C types
n_2	t_2	p_{j_2}	$= 10$	
$\vdots$	$\vdots$	$\vdots$	$= 15$	
n_c	t_c	p_{j_c}	$\vdots$	

$$\sum_{i=1}^{c} n_i = n$$

Suppose we also know the optimal makespan $= T^*$

$f(n_1', n_2', \ldots n_c')$ = min # of machines st we can schedule all jobs with load $\leq T^*$ on all machines.

$\downarrow$

$\boxed{\text{If } f(n_1, n_2, \ldots n_c) \leq m}$

Call $(l_1, l_2, \ldots l_c)$ valid configuration

If $\boxed{\sum l_i \cdot p_i \leq T^*}$

$f(0, 0, \ldots 0) = 0$

$1 + f(n_1 - l_1, n_2 - l_2 \ldots$

$$f(n_1', n_2', \ldots, n_c') = \min_{\substack{\text{all valid} \\ \text{conf } (l_1, l_2, \ldots l_c)}} 1 + f(n_1' - l_1, n_2' - l_2 \ldots n_c' - l_c)$$

Total # possible configurations

$$= (n_1+1)(n_2+1) \cdots (n_c+1)$$

$$\leq n^c$$

Total # valid configurations $\leq n^c$

Overall run-time $= O(n^{2c})$